Sudoku!

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What is Sudoku?

- a (very addictive) puzzle presented on a square grid that is usually 9×9, but may be other sizes
- invented by Howard Garns in 1979, and first published by Dell Magazines
- first became popular in Japan: “Sudoku” means “the digits must remain single”

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2		5			7			6
4			9	6			2	
				8			4	5
9	8			7	4			
5	7		8		2		6	9
			6	3			5	7
7	5			2				
	6			5	1			2
3			4			5		8

The original grid has some of the squares filled with the digits from 1 to 9. Each row, column, and 3×3 block must contain exactly the digits 1 to 9.

2		5			7			6
4			9	6			2	
				8			4	5
9	8			7	4			
5	7		8		2		6	9
			6	3			5	7
7	5			2				
	6			5	1			2
3			4			5		8

Using graph paper, see if you can solve this puzzle! (Provided on the worksheet).

2	3	5	1	4	7	9	8	6
4	1	8	9	6	5	7	2	3
6	9	7	2	8	3	1	4	5
9	8	6	5	7	4	2	3	1
5	7	3	8	1	2	4	6	9
1	4	2	6	3	9	8	5	7
7	5	9	3	2	8	6	1	4
8	6	4	7	5	1	3	9	2
3	2	1	4	9	6	5	7	8

Solution!

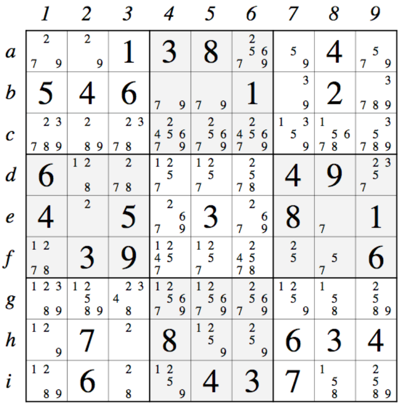
	2	4	3	8				
					6			7
	5	8				4		
4				1				
			7		5			
				2				8
		1				6	7	
3			5					
				4	9	2	1	

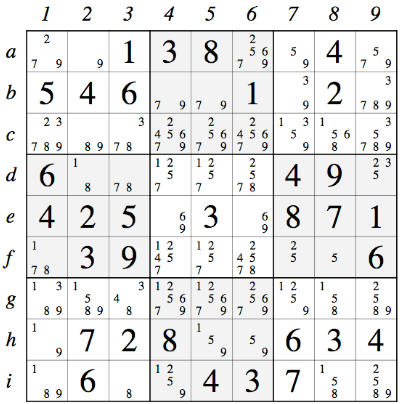
Now try this “medium-grade” puzzle.

6	2	4	3	8	7	5	9	1
1	3	9	4	5	6	8	2	7
7	5	8	1	9	2	4	3	6
4	9	6	8	1	3	7	5	2
2	8	3	7	6	5	1	4	9
5	1	7	9	2	4	3	6	8
9	4	1	2	3	8	6	7	5
3	6	2	5	7	1	9	8	4
8	7	5	6	4	9	2	1	3

Solution!

After some elimination, certain squares have only one candidate remaining. These are “naked singles”:





Notice that in column 2, the only square than can be 5 is $g2$. This is a “hidden single". Can you find another one?

	1	2	3	4	5	6	7	8	9
a	2 7 9	2 9	**1**	**3**	**8**	2 5 6 7 9	5 9	**4**	5 7 9
b	**5**	**4**	**6**	7 9	7 9	**1**	3 9	**2**	3 7 8 9
c	2 3 7 8 9	2 8 9	2 3 7 8	2 4 5 6 7 9	2 5 6 7 9	2 4 5 6 7 9	1 3 5 9	1 5 6 7 8	3 5 7 8 9
d	**6**	1 2 8	2 7 8	1 2 5 7	1 2 5 7	2 5 7 8	**4**	**9**	2 3 5 7
e	**4**	2	**5**	2 6 7 9	**3**	2 6 7 9	**8**	7	**1**
f	1 2 7 8	**3**	**9**	1 2 4 5 7	1 2 5 7	2 4 5 7 8	2 5	5 7	**6**
g	1 2 3 8 9	1 2 5 8 9	2 3 4 8	1 2 5 6 7 9	1 2 5 6 7 9	2 5 6 7 9	1 2 5 9	1 5 8	2 5 8 9
h	1 2 9	**7**	2	**8**	1 2 5 9	2 5 9	**6**	**3**	**4**
i	1 2 8 9	**6**	2 8	1 2 5 9	**4**	**3**	**7**	1 5 8	2 5 8 9

	1	2	3	4	5	6	7	8	9
a	2 7 9	9	**1**	**3**	**8**	2 5 6 7 9	5 9	**4**	5 7 9
b	**5**	**4**	**6**	7 9	7 9	**1**	3 9	**2**	3 7 8 9
c	2 3 7 8 9	8 9	3 7 8	2 4 5 6 7 9	2 5 6 7 9	2 4 5 6 7 9	1 3 5 9	1 5 6 8	3 5 7 8 9
d	**6**	1 8	7 8	1 2 5 7	1 2 5 7	2 5 7 8	**4**	**9**	2 3 5
e	**4**	**2**	**5**	6 9	**3**	6 9	**8**	**7**	**1**
f	1 7 8	**3**	**9**	1 2 4 5 7	1 2 5 7	2 4 5 7 8	2 5	5	**6**
g	1 3 8 9	1 5 8 9	3 4 8	1 2 5 6 7 9	1 2 5 6 7 9	2 5 6 7 9	1 2 5 9	1 5 8	2 5 8 9
h	1 9	**7**	**2**	**8**	1 5 9	5 9	**6**	**3**	**4**
i	1 8 9	**6**	8	1 2 5 9	**4**	**3**	**7**	1 5 8	2 5 8 9

Notice that the *def*789 block must contain a 2. Thus no other square in row *f* outside that block contains a 2. This forces *f*5 to be 3. The 2 is a “locked candidate”, locked inside block *def*789.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
a	1	4 5	8	6	7	2	4 5	9	3
b	3 5 7	3 5 7	9	8	1	4	6	5 7	2
c	2 4 7	6	2 7	9	5	3	8	4 7	1
d	3 4 5 9	3 4 5	6	2 4 5	2 3	7	1	8	5 9
e	4 5	2	1	4 5	9	8	7	3	6
f	3 4 5 7 9	8	3 5 7	1	2 3	6	2 4 5	2 4 5	5 9
g	2 5	1	4	3	8	9	2 5	6	7
h	6	3 5 7	2 3 5 7	2 7	4	1	9	2 5	8
i	8	9	2 7	2 7	6	5	3	1	4

What can be said about the row below?

	1	2	3	4	5	6	7	8	9
a	1	2 7	2 3 4	5	3 4 6	8	9	2 7	2 3 4 6 7

The 2 and 7 in squares *a*1 and *a*8 below are a “naked pair”. The 2 and 7 must be placed in those two squares. This allows us to eliminate 2 from *a*3, and both 2 and 7 from *a*9.

	1	2	3	4	5	6	7	8	9
a	1	2 7	2 3 4	5	3 4 6	8	9	2 7	2 3 4 6 7

What can be said about the row below?

	1	2	3	4	5	6	7	8	9
a	5	1 7	2	1 4 8	1 3 4 8	9	6	3 7	1 3

The 1,3, and 7 in squares *a*2, *a*8, and *a*9 below are a “naked triple”. Those 3 numbers must go in those three squares. This allows us to eliminate 1 from *a*4, and both 1 and 3 from *a*5.

	1	2	3	4	5	6	7	8	9
a	5	1 7	2	1 4 8	1 3 4 8	9	6	3 7	1 3

What can be said about the row below?

i	1 2 4 6	3 9	2 6	5	1 3 8	4 8	2 7	2 7 9	3 6

The 1,4, and 8 in squares *i*1, *i*5, and *i*6 below are a “hidden triple”. Those 3 numbers must go in those three squares. This allows us to eliminate 2 and 6 from *i*1, and 3 from *i*5.

i	1 2 4 6	3 9	2 6	5	1 3 8	4 8	2 7	2 7 9	3 6

	1	2	3	4	5	6	7	8	9
a			7				3	2	
b					1	9		7	
c			2		7	3	6		9
d				1					7
e			6				4		
f	4					8			
g	2		4	5	6		9		
h		5		9	8				
i		9	1				7		

Now try this puzzle, which focuses on naked triples.

What's going on with the 4 squares circled below?

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
a	1 2 6	4	8	7	9	3 6	1 2 3 5	1 2 3 5	2 3
b	1 6 9	5	1 3	8	2	3 6	7	1 3 4	3 4 9
c	2 9	2 3 9	7	5	4	1	2 3 9	6	8
d	3	8	5	2	1	9	4	7	6
e	7	6	2	3	5	4	8	9	1
f	4	1	9	6	7	8	2 3	2 3	5
g	8	7	6	4	3	5	1 2 9	1 2	2 9
h	1 5 9	3 9	4	1 9	6	2	3 5	8	7
i	1 2 5 9	2 3 9	1 3	1 9	8	7	6	3 4 5	3 4

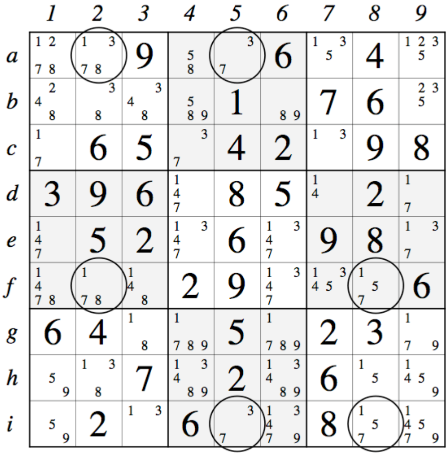
The 3 must be in *c*2 and *h*7, or *h*2 and *c*7. This is an “X-wing” configuration.

	1	2	3	4	5	6	7	8	9
a	1 2 6	4	8	7	9	3 6	1 2 3 5	1 2 3 5	2 3
b	1 6 9	5	1 3	8	2	3 6	7	1 3 4	3 4 9
c	2 9	2 3 9	7	5	4	1	2 3 9	6	8
d	3	8	5	2	1	9	4	7	6
e	7	6	2	3	5	4	8	9	1
f	4	1	9	6	7	8	2 3	2 3	5
g	8	7	6	4	3	5	1 2 9	1 2	2 9
h	1 5 9	3 9	4	1 9	6	2	3 5	8	7
i	1 2 5 9	2 3 9	1 3	1 9	8	7	6	3 4 5	3 4

What's going on with the circled squares below?

	1	2	3	4	5	6	7	8	9
a	1 2 7 8	1 3 7 8	9	5 8	3 7	6	1 3 5	4	1 2 3 5
b	2 4 8	3 8	3 4 8	5 8 9	1	8 9	7	6	2 3 5
c	1 7	6	5	3 7	4	2	1 3	9	8
d	3	9	6	1 4 7	8	5	1 4	2	1 7
e	1 4 7	5	2	1 3 4 7	6	1 3 4 7	9	8	1 3 7
f	1 4 7 8	1 7 8	1 4 8	2	9	1 3 4 7	1 3 4 5	1 5 7	6
g	6	4	1 8	1 7 8 9	5	1 7 8 9	2	3	1 7 9
h	5 9	1 3 8	7	1 3 4 8 9	2	1 3 4 8 9	6	1 5	1 4 5 9
i	5 9	2	1 3	6	3 7	1 3 4 7 9	8	1 5 7	1 4 5 7 9

7 can be eliminated from *a*1, *f*1, *f*6, *i*6, and *i*9. This is a "Swordfish" configuration.



Interesting facts:

- There are 5524751496156892842531225699 Latin squares that are 9×9, which are Sudoku puzzles without the constraint that 3×3 blocks contain all the digits.
- There are 6670903752021072936960 Sudoku puzzles.
- There are 18383222420692992 *inequivalent* Sudoku puzzles.
- There are 4752 *magic* Sudoku puzzles, which are Sudoku puzzles where each diagonal contains all the digits.

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Unsolved Problem:

What is the minimal number of locations that must be filled in an otherwise empty grid that will guarantee there is a unique solution to the puzzle? Examples exist of puzzles with unique solutions that have only 17 locations filled initially.

Discussion questions:

- What have we learned here?
- How could any of this be used in the classroom?

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